# Lateral Groundwater Flow Equation

$$\frac{\partial}{\partial x}\left(K_{xx} b \frac{\partial E_{ws}}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_{yy} b \frac{\partial E_{ws}}{\partial y}\right)=S\frac{\partial E_{ws}}{\partial t}+W(x,y,t)$$

Free Surface lateral groundwater flow in 2D

- *K* is the saturated hydraulic conductivity
- *b* is the depth of water in the aquifer
- *E* is the water surface elevation
- *S* is the storage term
- *W* is the source/sink term

What does it tell us?

The change in head in space is equal to the change in head in time plus source/sinks.

# Solution

- The free surface groundwater flow equation is solved for heads.
- The discretization is 5 point implicit finite difference.

$$\frac{1}{\Delta x_j}\left[\left(T_{xx(i,j+1/2,k)}\frac{h_{i,j+1}^{n+1}-h_{i,j}^{n+1}}{\Delta x_{j+1/2}}\right)-\left(T_{xx(i,j-1/2,k)}\frac{h_{i,j}^{n+1}-h_{i,j-1}^{n+1}}{\Delta x_{j-1/2}}\right)\right]$$
$$+\frac{1}{\Delta y_i}\left[\left(T_{yy(i+1/2,,k)}\frac{h_{i+1,j}^{n+1}-h_{i,j}^{n+1}}{\Delta y_{i+1/2}}\right)-\left(T_{yy(i-1/2,j,k)}\frac{h_{i,j}^{n+1}-h_{i-1,j}^{n+1}}{\Delta y_{i-1/2}}\right)\right]$$
$$=\frac{S_{i,j}^{n+1}}{\Delta t}\left(h_{i,j}^{n+1}-h_{i,j}^{n}\right)+W_{i,j}^{n+1}$$

- The solution is by Linear Successive Over Relaxation (LSOR).
- For each time step the model iterates on the heads until a closure criteria is achieved.
- The storage and transmissivity terms are calculated for each iteration.

# Stream Boundary Conditions

- No special boundary condition
- Head boundary condition – Type 5
  - Head in the stream cell at the time of the groundwater update is imposed as a head boundary condition on the groundwater solution
- River flux boundary condition – Type 4
  - For positive flux (flow from groundwater to the stream)

$$f = -\frac{K_{rb}}{M_{rb}}\left(E_r - E_{ws}\right)$$

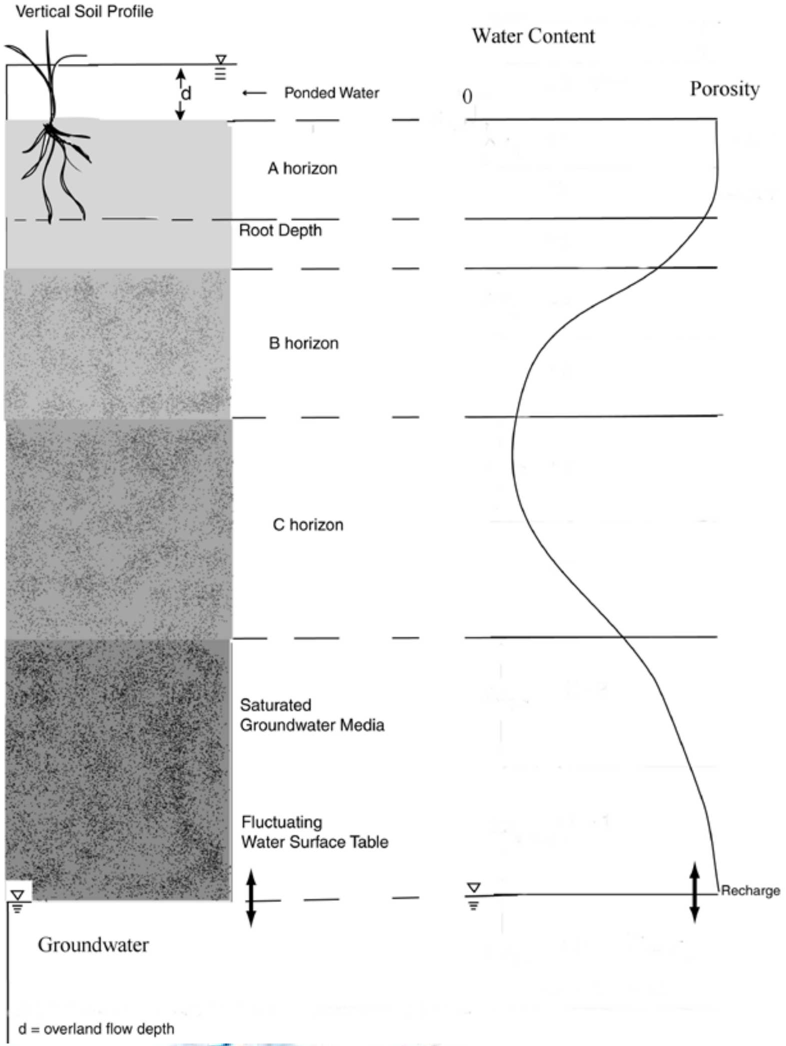
where: $f$ = per unit area discharge (m s-1),
$K_{rb}$ = hydraulic conductivity of the river bed material (cm $hr^{-1}$),
$M_{rb}$ = depth of the river bed material (cm),
$E_r$ = elevation of the river water surface (m), and
$E_{ws}$ = elevation of the groundwater water surface (m).

# Stream Boundary Condition

- River flux when stream discharging to groundwater
  - $f = -K_{rb}$
- User must specify $M_{rb}$ and $K_{rb}$
  - M_RIVER (cm)
  - K_RIVER (cm $hr^{-1}$)
  - Specify same value for every stream cell in the project file
  - Specify values for every stream link in the channel input file (*.cif)
- Typically $M_{rb}$ set to 1.0 and $K_{rb}$ is a calibration parameter

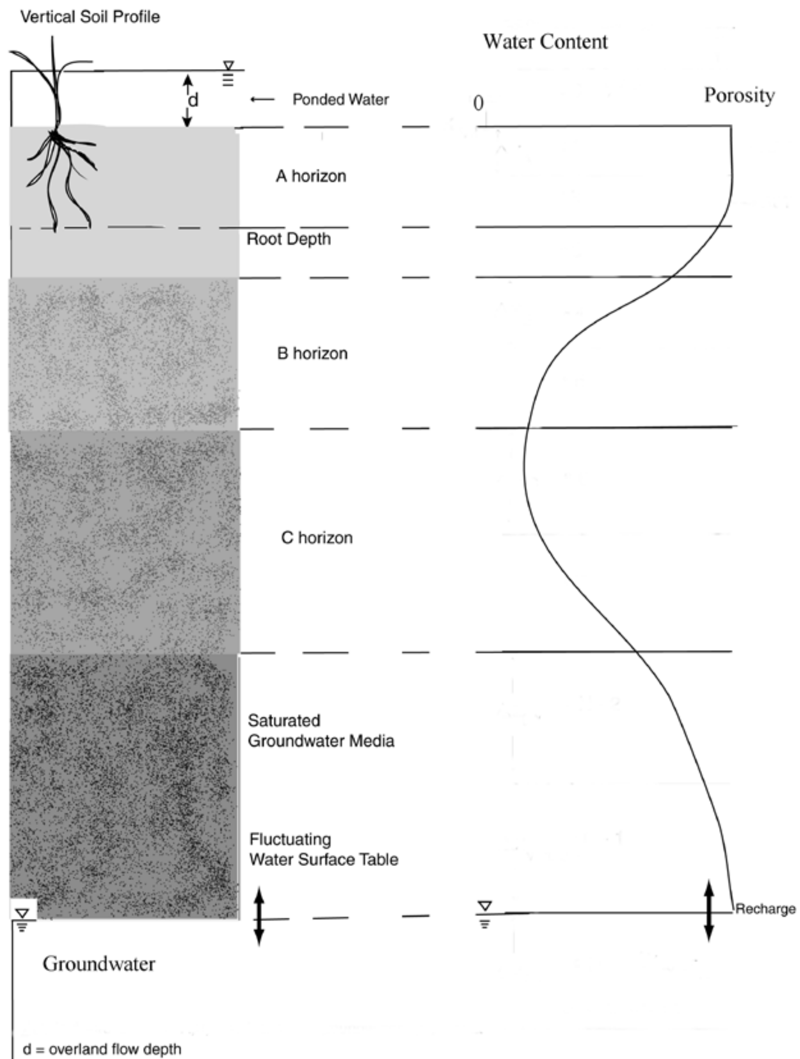
# Coupling with Richards Equation

- Soil moisture is calculated in discrete increments from the soil surface to the water table.
- Soil profile “floats” on top of the saturated groundwater surface.
- The number of cells in the profile varies as the water table rises and falls.
- 4 levels of discretization are specified
    - 1st Soil layer
    - 2nd Soil layer
    - 3rd Soil layer
    - Unsaturated media between bottom of third soil layer and the groundwater surface.



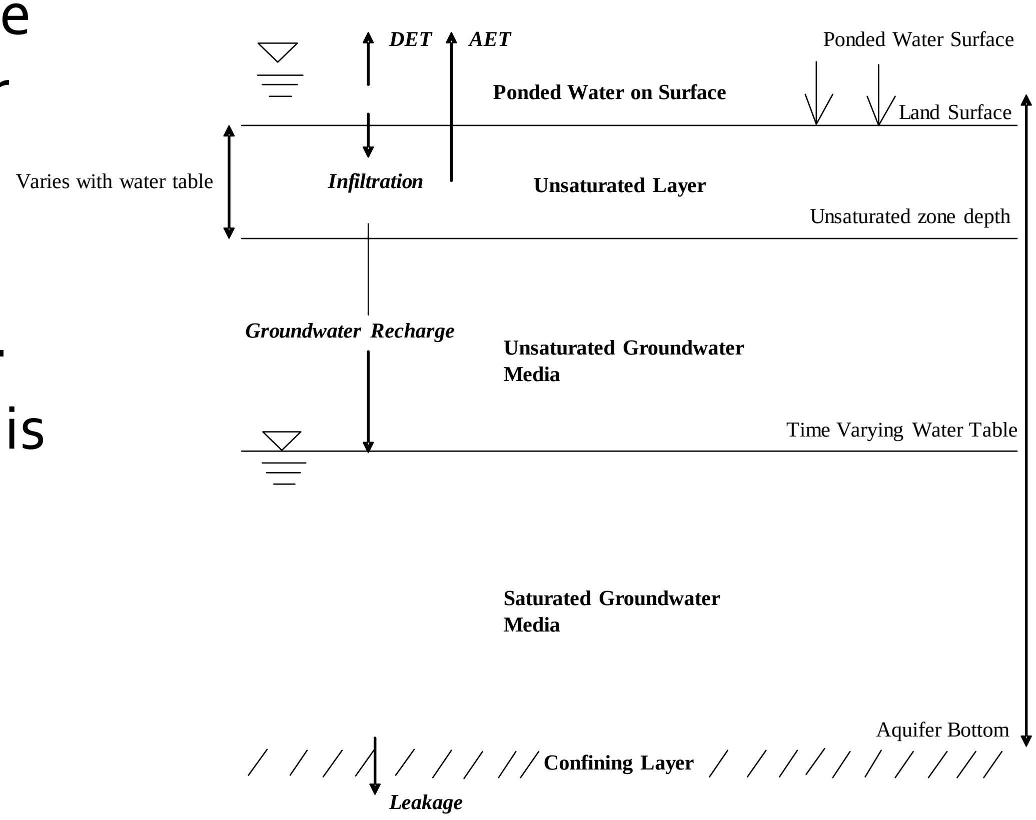
# Coupling to Richards Equation

- Solution of the RE integrates infiltration, ET, soil water movement, soil moistures and the lower boundary condition.
  - Infiltration and ET feel the effects of the groundwater location
- Movement of water is due to both gravity and matric potential, so water flows both to and from the groundwater, depending on soil moisture conditions.
  - To the groundwater during wet periods of recharge
  - From the groundwater during dry periods of high ET
  - Flux to the groundwater is affected by entire soil moisture profile, as well as the upper boundary conditions, infiltration and ET.



# Coupling to G&A Methods

- The saturated groundwater model can also be linked to the surface with either the GAR or MLGA models.
- In either case the effective coupling is basically the same.
- Recharge to the groundwater is computed as the flux of water from the bottom of the two layer soil model.
- There is no ET extraction of groundwater



# Simplifications in Relation to Richards

- No timing delays or flux changes due to area between bottom of soil layer and surface of groundwater.
- Single, static, soil moisture for layer between soil layer and surface of groundwater.
- No upward movement of water from groundwater.
- No effect of groundwater on infiltration unless groundwater level rises into soil column.