Watershed Scale TMDL Model: Multidimensional Sediment Erosion, Transport, and Fate

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PURPOSE: The purpose of this System-Wide Water Resources Program (SWWRP) technical note is to describe the new sediment transport routines in GSSHA, a watershed analysis and management tool that has the ability to simulate the movement of water, sediment, and associated constituents at fine-scale increments (<100 m) over watershed scale areas. The resulting tool is intended for analyzing project alternatives and best management practices (BMPs) designed to control sediments near the source, either on upland areas or in tributaries.

BACKGROUND: Receiving water bodies are harmed by the introduction of excess sediments which reduce storage capacity, increase turbidity, and introduce associated contaminants. The control of sediments may best be performed in upland areas near their source where contaminants may be removed along with the sediments before the contaminants are released into solution.

In this work unit, the Department of Defense Watershed Modeling System (WMS)-distributed hydrologic model Gridded Surface Subsurface Hydrologic Model (GSSHA) (Downer et al. 2005) has been further developed to allow the physics based simulation of sediment erosion, transport, and deposition on a continuous basis. Simulation of sediments is at the most fundamental level of current understanding of sediment physics. The general laws of conservation of mass and momentum are applied. This approach allows contaminants closely associated with sediments to be simulated with the same approach and equations. A continuous simulation model provides the ability to track sediments over several precipitation events, or years, and determine the long-term patterns of erosion, deposition, and geomorphologic changes within a watershed.

GSSHA is a distributed, physics-based hydrologic model developed to simulate a watershed's response to meteorological inputs. Basic simulated physical processes include distributed rainfall, rainfall interception by vegetation, surface ponding and retention, infiltration, evapotranspiration, overland flow, streamflow, and lateral saturated groundwater flow. The model also simulates subsurface drainage networks and includes seasonality effects. The model is intended for, and has been applied to, the simulation of streamflow, flooding, soil moisture, sediment erosion, and discharge.

The hydrologic and hydraulic components of GSSHA provide the information necessary to simulate erosion and sediment fate and transport. Key to simulating sediments within a watershed, GSSHA provides the ability to link two-dimensional (2-D) overland flow transport to one-dimensional (1-D) transport in a stream network as shown in Figure 1. The overland flow plane provides inputs to the channel network at each point where the channel network and overland flow plane coincide. Optionally, water and constituents can be allowed to spill back on the overland flow plane.

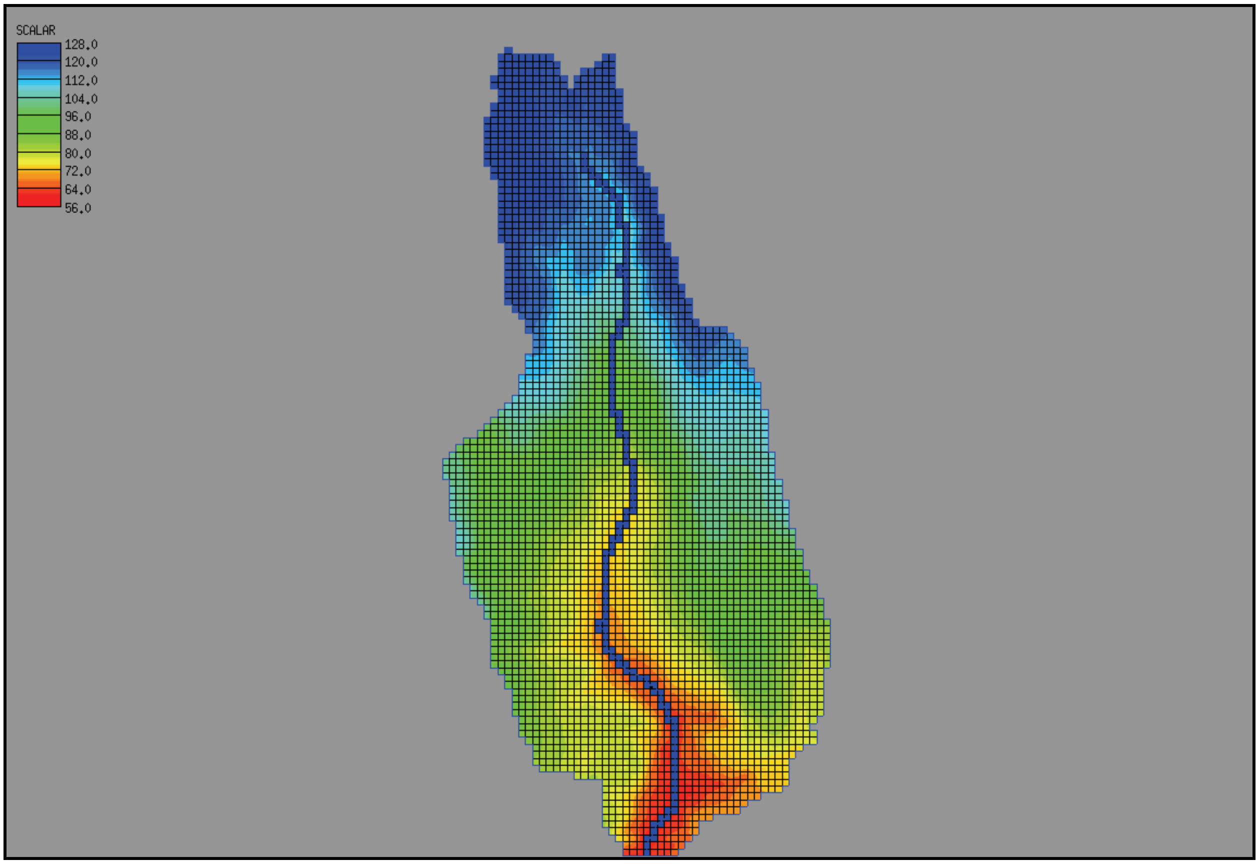

Figure 1. Representation of watershed and simple stream network in GSSHA.

Flow in both domains is described by the diffusive wave approximation of the de Saint Venant equations and solved using the finite volume technique. The finite volume method allows the simultaneous simulation of both wet and dry cells. The ability to simulate "dry bed" conditions is critical in the simulation of large regional watersheds as precipitation may occur preferentially within the study area and only parts of the watershed, or stream network, may be flowing at any given time.

Prior to this effort, GSSHA employed an empirical approach to sediment erosion, transport, and fate (Johnson et al. 2000; Sanchez 2002) taken from the CASC2D model (Ogden and Julien 2002), GSSHA's predecessor. Due to limitations of this method (Ogden and Heilig 2001) and a desire to simulate sediments and dissolved constituents with one general method, the more empirical method has been replaced with a more general, physics-based approach as described in this document.

METHODOLOGY: The general transport equations are solved in two dimensions on the overland flow plane and in one dimension in the channel network, using a common methodology and interaction between the two domains. Sediment erosion and deposition are treated as first order reactions, with erosion being an source term, and deposition being a sink term.

GENERAL TRANSPORT EQUATIONS: The general transport equations describe the fate and transport of dissolved and particulate constituents. Each cell is treated as a completely mixed reactor and the concentration in each cell is affected by internal sources and sinks, as well by advection into and out of the cell and diffusion between surrounding cells.

For the sake of simplicity, the 1-D scheme (channels) is presented first. Although only one constituent is described in this text, the number of constituents, both dissolved and particulate (sediment), can be any number specified by the user.

Reactive Transport in Channels. GSSHA computes values of water flow and depth within a user-specified 1-D finite volume stream network. Solution of the diffusive wave approximation provide the discharge and cross-sectional area at fine space and time increments within the channel network. Typical stream nodes range in size from 30 to 200 m. Typical channel routing time-steps range from 5 to 30 sec.

This information is needed for the transport of sediments and other constituents within the channel network using the general 1-D advection-dispersion equation in terms of the mass of constituent (M) equal to the concentration (C) multiplied by the volume (V) with constant dispersion:

$$\frac{\partial(M)}{\partial t} + \frac{\partial(uM)}{\partial x} - \frac{\partial}{\partial x}\left(D\frac{\partial M}{\partial x}\right) + K(M) = S \tag{1}$$

where:

u = flow velocity in the x-direction (M T^{-1})
C = concentration (M L^{-3})
Q_x = discharge in the x-direction (L^3 T^{-1})
A = cross-sectional area (L^2)
D_x = diffusion coefficient in the x-direction for the constituent of concern (L^2 T^{-1})
K = decay coefficient (T^{-1})
V = volume (L^3)
S = term for all sources and sinks (M T^{-1})

This equation is solved with a simple three-point explicit finite-volume scheme for each constituent of concern. The solution proceeds from the most upstream node and proceeds downstream to the channel outlet. This scheme is first order accurate in time and space. For each node, the following mass reactions are accounted for with masses (M) in grams; volumes (V) in m^3; discharges (Q) in m^3 s^{-1}; and concentrations (C) in g m^{-3} or mg L^{-1}.

- Initial mass, $M_o = \Delta x(C_i^n A_i^n)$
  - Where A is the cross-sectional area, Δx is the channel grid element size, the superscript n refers to the current time level, and the subscript i refers to the current node.
- Advective flux, $M_{advect} = \Delta t\left(Q_{i-1}^{n+1} C_{i-1}^n - Q_i^{n+1} C_i^n\right)$,
  - Where Δt is the time-step (s); the subscripts i-1, and i refer to upstream node, and current node, respectively, with flow being defined as positive from the current cell to the downstream cell; the superscript n+1 refers to the next time level.
- Constant point source, $M_p = \Delta t\left(Q_p C_p\right)$
  - Where Q_p and C_p are the flow and concentration of the point source, respectively.
- Lateral flow, exchange between channel and overland flow plane, $M_{lat} = \Delta t \Delta l\left(q_{lat}^{n+1} C_{lat}^n\right)$.
  - Where Δl is length of the stream/overland interface (m), and C_{lat} is the concentration of the water on the overland plane if flow is into the channel, and the concentration of the water in the channel if flow is back onto the overland plane.
- Groundwater exchange, $M_{gw} = \Delta t\left(Q_{gw}^{n+1} C_{gw}^n\right)$
  - Where the concentration of the groundwater, C_{gw}, depends on the direction of Q. It is equal to C_i if the flux is into the groundwater (Q is negative) or equal to concentration in the groundwater if the flux is into the stream (Q is positive).
- Dispersive exchange, $M_{disp} = \dfrac{\Delta t}{\Delta x}\left(D_{i+1/2} A_{i+1/2}^{n+1}(C_{i+1}^n - C_i^n) - D_{i-1/2} A_{i-1/2}^{n+1}(C_i^n - C_{i-1}^n)\right)$
  - Where: D is the dispersion coefficient (m^2 s^{-1}), assuming D varies in space, but not in time, and $A_{i+1/2}$ and $A_{i-1/2}$ are the cross-sectional areas (m^2) between the i and $i+1$ and i and $i-1$ nodes, respectively.
- Decay, $M_{decay} = \Delta t\left(K_{decay}^{n+1}\ C_i^n V^n\right)$
  - Where K_{decay} is the decay coefficient computed at each time increment based on the physical state of the sytem (s^{-1}).

Concentration at the next time level, $n+1$, is calculated as:

$$C_i^{n+1} = \frac{M_o + M_{advect} + M_p + M_{lat} + M_{gw} + M_{disp} + M_{decay}}{V_i^{n+1}} \qquad (2)$$

The mass exchange between the channel and the groundwater is only computed if saturated groundwater computations are being conducted. The concentration in the groundwater is assumed to be zero, so that $C_{gw} = 0$ when the stream is gaining. The concentration of the overland flow plane, C_{lat}, will also be zero unless the overland flow transport is also being simulated.

Overland Reactive Transport. For overland flow, the 2-D form of the general transport equation is solved.

$$\frac{\partial(M)}{\partial t}+\frac{\partial(uM)}{\partial x}+\frac{\partial(vM)}{\partial y}-\frac{\partial}{\partial x}\left(D_x\frac{\partial M}{\partial x}\right)-\frac{\partial}{\partial y}\left(D_y\frac{\partial M}{\partial y}\right)+KM=S \qquad (3)$$

Where v is the velocity in the y-direction (m s^{-1}).

A 2-D overland flow solution of the diffusive wave approximations, as described by Downer and Ogden (2004), provides the necessary discharges and areas. As with the instream contaminant transport routines, the overland flow transport is modeled with a mass balance in each overland flow cell. For each constituent and each cell, the following mass reactions are accounted for with masses (M) in grams; volumes (V) in m^3; discharges (Q) in m^3 s^{-1}; and concentrations (C) in g m^{-3} or mg L^{-1}.

- Initial mass, $M_o = \Delta t \Delta x^2 C_{ij}^n h_{ij}^n$
  - Where: Δx is the grid size (m), h is depth, and the subscript ij refers x and y location, respectively, of the current node.
- Precipitation Mass, $M_{pre} = \Delta t \Delta x^2 (IC_{pre})$
  - Where I is rainfall intensity (m s^{-1}) and C_{pre} is the concentration in the precipitation (assumed to be zero).
- Advective flux in the x-direction, $M_x = \Delta t \Delta x \left(p_{i-1,j}^{n+1} C_{i-1,j}^n - p_{i,j}^{n+1} C_{i,j}^n\right)$
  - Where p is the unit discharge (uh) in the x-direction (m^2 s^{-1}).
- Advective flux in the y-direction, $M_y = \Delta t \Delta x \left(q_{i,j-1}^{n+1} C_{i,j-1}^n - q_{i,j}^{n+1} C_{ij}^n\right)$
  - Where: q is the unit flux (vh) in the y-direction (m^2 s^{-1}).
- Point source, $M_p = \Delta t \left(Q_p C_p\right)$,
- Lateral inflow to channel, $M_{lat} = -\Delta t \Delta l \left(q_{lat}^{n+1} C_{lat}^n\right)$
- Infiltration loss, $M_{inf} = \Delta t \Delta x^2 \left(f_{inf}^{n+1} C_{ij}^n\right)$
  - Where: f_{inf} is the infiltration rate (m s^{-1}).
- Exfiltration, $M_{exf} = \Delta t \Delta x^2 (f_{exf}^{n+1} C_{gw})$
  - Where: f_{exf} is the exfiltration rate (m s^{-1}) and C_{gw} is assumed to be constant.
- Dispersive exchange in x-direction,

  $$M_{disp_x} = \frac{\Delta t}{\Delta x^2}\left(D_{i+1/2,j} A_{i+1/2,j}^{n+1} (C_{i+1,j}^n - C_{ij}^n) - D_{i-1/2,j} A_{i-1/2,j}^{n+1} (C_{ij}^n - C_{i-1,j}^n)\right),$$

- Dispersive exchange in the y-direction,
  $$M_{disp_y} = \frac{\Delta t}{\Delta x^2}\left(D_{ij+1/2}A^{n+1}_{i,j+1/2}(C^n_{i,j+1} - C^n_{ij}) - D_{i,j-1/2}A^{n+1}_{i,j-1/2}(C^n_{ij} - C^n_{i,j-1})\right)$$

- Decay, $M_{decay} = \Delta t(M_o K^{n+1}_{decay})$

- Uptake from the land surface, $M_{uptake} = K^{n+1}_{uptake}(C_{\max} - C^n_{ij})\Delta t \Delta x^2$
  - Where: K_{uptake} is the uptake coefficient (g m^{-2} s^{-1}) and C_{max} is the maximum possible concentration in the overland flow cell.

The 2-D overland transport equations are solved with a five-point explicit alternating direction (ADE) scheme with prediction correction (PC), or ADE-PC (MacCormick 1971). This method is 2nd order accurate in space and time. The concentration at the next time-step, *n+1* time level, is computed using the following method.

- If the overland cell is located along a channel, the volume, mass, and concentration in that cell is first corrected for lateral inflow into the channel node or channel flow back on the overland flow plane.

- The volume, mass, and concentration are then adjusted for precipitation, point sources, and exfiltration.

- Uptake and decay rates are calculated in each node, solids only, as described in the next section.

- Infiltration, dispersion, decay and advection in the x- and y-directions are computed using an alternating direction explicit ADE-PC, which proceeds in the following manner.
  - Changes in cell volume, mass and concentration due to infiltration, decay, and advection and dispersion in the y-direction are first computed using values at the *n* time level.
  - Intermediate values of volume and concentration at the *n+1/2* time level are computed based on the mass changes at the *n* time level.
  - An average of the values of volumes and concentrations at the *n* time level and the intermediate values at the *n+1/2* time level are used to compute new values of concentration at the *n+1/2* time level.
  - Changes in mass and concentration due to infiltration, decay, and advection and dispersion in the y-direction are computed using the new values of concentration at the *n+1/2* time level.
  - Final values of concentration at the *n+1/2* time level are computed based on the final mass fluxes computed at the *n+1/2* time level.
  - The preceding steps are repeated for the x-direction, yielding values of concentration at the *n+1* time level. To avoid any directional bias, the order of solution, first *y* and then *x*, or first *x* and then *y*, changes every time-step.

When solids are being transported, there is no loss or gain of the solids as water infiltrates, evaporates, rains, or exfiltrates, but the concentration of solids in the cell is modified in response to the reduction in the volume of water in the cell.

Uptake – Erosion. Though the physics of erosion on the overland flow plane are not well described, noncohesive particles are thought to erode due to the hydraulic properties of flow and rainfall impact. However, in order to test the overall model formulation, some currently applied methods of computing overland erosion are incorporated into the model.

In overland flow simulations, the erosion is typically described in terms of the transport capacity, the amount of sediment that can be transported under the given hydraulic conditions. As reviewed by Julien and Simons (1985), transport capacities are typically of the form of the original Kilinc-Richardson equation (Kilinc and Richardson 1973):

$$q_s = 25500 q^{2.035} S_o^{1.664} \tag{4}$$

where the factor 25,500 is an empirical constant, q_s is the sediment unit discharge (ton m^{-1} s^{-1}) q is unit discharge (m^2 s^{-1}) and S_o is the land surface slope.

Julien (1995) modified the original Kilinc-Richardson equation to allow the transport capacity to be computed with conditions of nonuniform flow with consideration of soil and land-use specific factors to compute the transport capacity (tons m^{-1} s^{-1}):

$$q_s = 25500 q^{2.035} S_f^{1.664} \frac{E * C * P}{0.15} \tag{5}$$

where:

q = unit discharge (m^2 s^{-1})
S_f = friction slope (dimensionless)
E = soil erodability factor, with values ranging from 0 to 1
C = soil cropping factor (0-1)
P = conservation factor (0-1)

In CASC2D-SED (Johnson et al. 2000; Sanchez 2002) and previous versions of GSSHA, this transport capacity is computed in both the x- and y-directions and then used to transport deposited and parent materials, proportional to available, in each direction. The transport capacity is always satisfied.

For inclusion into the preceding general transport equations, the transport capacity must be changed into an erosion coefficient. Because determining the interdependent factors E, C, and P is difficult (Ogden and Helig 2001), they are combined into a single coefficient, F_{erode}, representing the overall erodability of soils. The x and y components of both unit discharge, q, and friction slope, S_f, are used to compute the magnitude of total transport capacity in the Kilinc-Richardson equation. When divided by the grid size, Δx, an uptake rate, K_{uptake}, (g m^{-2} s^{-1}) is provided to be used in the general transport equations. The final equation is:

$$K_{uptake} = \frac{25500}{1.102x10^{-6}} q^{2.035} S_f^{1.664} \frac{F_{erode}}{0.15\Delta x} \tag{6}$$

Alternately the uptake rate for each particle class being simulated may be computed from the Engelund-Hansen equation (Engelund and Hansen 1967) as:

$$K_{uptake} = \frac{0.05 F_{erode} \gamma_s v^2}{\Delta x} \left[\frac{d_{50}}{g(\frac{\gamma_s}{\gamma_f} - 1)} \right]^{1/2} \left[\frac{\tau_0}{(\gamma_s - \gamma_f\ ) d_{50}} \right]^{3/2} \tag{7}$$

where

v = magnitude of flow velocity (m s^{-1})
d_{50} = median particle size for each class specified (m)
g = acceleration of gravity (m s^{-2})
γ_f = specific weight of the fluid (N m^{-3})
γ_s = specific weight of the particle (N m^{-3})
τ_0 = bed shear stress (N m^{-2}), computed as:

$$\tau_0 = \gamma_f h S_f \tag{8}$$

where: h is the depth of water in the cell of interest (m) which may range from less than a centimeter in areas of sheet flow to perhaps one meter in concentrated flow areas. The velocity in the cell can be computed from the x and y components of unit discharge as:

$$v = \sqrt{\left(\frac{q}{h}\right)^2 + \left(\frac{p}{h}\right)^2} \tag{9}$$

According to Govers (1990), transport does not occur unless the stream power, Ω, is greater than 0.004 m s^{-1}. Therefore, for the Engelund-Hansen equation, $K_{uptake} = 0$ for $\Omega < 0.004$. The stream power is computed as:

$$\Omega = v S_f \tag{10}$$

Erosion due to rainfall impact is simulated as described by Dario (2002). When total erosion is calculated, as in the Kilinc-Richardson equation, the amount of erosion of each particle size is determined from the fractions of particles available in the parent material. For cells with deposited materials, these materials are eroded first, then any additional erosion that occurs is from the parent material. The distribution of particles in the deposited materials is likely different, and tracked separately, from the parent material. Particles are tracked according to size (as specified), location (which cell), and status (parent material, in suspension, or deposited).

Maximum Concentration. Uptake mass is limited by the maximum concentration that the fluid can transport. This can be taken as a concentration equal to the specific gravity of the particle times the water density, or computed from a variety of methods. As implemented in KINEROS (Woolhiser, Smith, and Goodrich 1990), the Engelund-Hansen equation can be used to determine the maximum concentration, C_{max}, (g m^{-3}) for each sediment as:

$$C_{\max} = \frac{0.05 SG \rho}{d_{50}\left(SG-1\right)^2}\sqrt{\frac{S_f h}{g}}\left(\Omega - 0.004\right) \tag{11}$$

where SG is the specific gravity of the particles, and ρ is the density of water at 20° C (g m^{-3}).

These equations are included because they have some general applicability and acceptance and some method of estimating erosion uptake rate and maximum concentration is needed to allow the overall method to be tested.

Decay – Deposition. Deposition is controlled by particle shape, size (d_{50}), specific gravity (SG), and the properties of the fluid that the particle is settling in. For particles larger than 1 mm, the settling velocity, v_s (m s^{-1}) is (Julien 1995):

$$\omega = \frac{8\nu}{d}\left[\sqrt{1+0.0139 d_*^3} - 1\right] \tag{12}$$

where

ν = kinematic viscosity of the fluid (m^2 s^{-1})
d = median particle size, d_{50} (m)
d_* = dimensionless particle diameter, computed as:

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$$d_* = d\left[\frac{(SG-1)g}{\nu^2}\right]^{1/3} \tag{13}$$

For particles smaller than 1 mm, the Stokes equation is used:

$$\omega = \frac{gd^2}{18\nu}\frac{\gamma_s - \gamma_f}{\gamma_f} \tag{14}$$

The kinematic viscosity of water depends on water temperature. For long-term simulations, where hourly values of air temperature are provided, overland water temperature is assumed to be equal to the air temperature. For single event simulations the water temperature is specified; 20° C is the default value.

In the general transport equations previously described, deposition is treated as decay. The decay rate K_{decay} (s^{-1}) is calculated each time-step as:

$$K_{decay} = \frac{v_s}{h} \tag{15}$$

REACTIVE TRANSPORT: Although the schemes described in this technical note were developed for transport of sediments, the techniques employed are general and the same methods could be used to simulate the fate and transport of dissolved reactive constituents. For simple first order reactions, the decay rates (K) would reflect decay rates for the loss of the reactive constituent. For complex higher order reactions with multiple subspecies and transformations between these species, the change in mass due to multiple reactions (KV) could be calculated outside the GSSHA model and passed back to the reactive transport routine. This approach would allow any number of subspecies to be accounted for without adding undue complexity to the basic GSSHA formulation.

SUMMARY: The GSSHA model has been modified to allow the transport and fate of constituents on both the overland flow plane and within the stream network with the general transport equations. The user specifies the number and properties of constituents to be simulated. The uptake of constituents from the overland flow plane and loss of materials in either the overland flow plane or stream network is simulated as first order reactions. For solids, sediments, erosion by any means, hydraulic or due to rainfall impact, is converted into an uptake rate. Deposition is calculated as a decay rate. These uptake and decay rates are computed in each grid, during each time-step. The method allows sediments to erode in one cell, be advected to downstream cells, and be deposited in either the cell of origin or in downstream cells. Erosion can be simulated for extended periods, allowing the trends in erosion and deposition, and resulting morphological changes in the watershed to be tracked. Ongoing research will identify the important processes in erosion, transport, and deposition providing improved estimates of erosion rates and sediment redistribution on the landscape.

ADDITIONAL INFORMATION: This SWWRP technical note was prepared by Dr. Charles W. Downer and Aaron Byrd, Coastal and Hydraulics Laboratory, U.S. Army Engineer Research and Development Center. The study was conducted as an activity of the Regional Sediment Management work unit of the System-Wide Water Resources Program (SWWRP). For information on SWWRP, please consult https://swwrp.usace.army.mil/ or contact the Program Manager, Dr. Steven L. Ashby at Steven.L.Ashby@erdc.usace.army.mil. This technical note should be cited as follows:

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